

N10K

**NASA TECHNICAL
MEMORANDUM**

NASA TM X-52291

NASA TM X-52291

FACILITY FORM 602	N 68-25 246	
	(ACCESSION NUMBER)	(THRU)
	12	1
	(PAGES)	(CODE)
	NASA-TMX-52291	25
	(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

THE SELECTION OF SIMPLE MODEL ATOMS
FOR CALCULATIONS OF ELECTRON DENSITY
IN NONEQUILIBRIUM, LOW TEMPERATURE Cs PLASMAS

by John V. Dugan Jr.
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at 27th Annual
Conference for Physical Electronics sponsored by
Massachusetts Institute of Technology
Cambridge, Massachusetts, March 20-22, 1967

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION - WASHINGTON, D.C. - 1967

~~CONFIDENTIAL~~
and Research Centers
only

THE SELECTION OF SIMPLE MODEL ATOMS FOR CALCULATIONS OF
ELECTRON DENSITY IN NONEQUILIBRIUM, LOW
TEMPERATURE Cs PLASMAS

by John V. Dugan Jr.

Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at
27th Annual Conference for Physical Electronics
sponsored by Massachusetts Institute of Technology
Cambridge, Massachusetts, March 20-22, 1967

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION



THE SELECTION OF SIMPLE MODEL ATOMS FOR CALCULATIONS OF ELECTRON DENSITY IN NONEQUILIBRIUM, LOW TEMPERATURE Cs PLASMAS

John V. Dugan Jr.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

ABSTRACT

E-3904

Free electron number densities N_e are calculated for cesium plasmas with low electron temperatures T_e (0.15 - 0.3 eV) using 5-, 3-, and 2-level atomic models. These N_e values are obtained for a Maxwellian distribution of free electrons; this distribution was shown to be a good approximation for $T_e > 2100^\circ \text{K}$ in optically thick Cs plasmas. The sensitivity of N_e to the choice of electronic levels, statistical degeneracy and radiative capture coefficients is presented for plasmas either optically thin or thick to resonance radiation. The steady state equation for the electron number density is discussed in the limits of low and high neutral density N_{Cs}^0 . The ionization fraction $f \equiv N_e/N_{\text{Cs}}^0$ was studied as a function of N_{Cs}^0 for the range of T_e from 1740°K (0.15 eV) to 3000°K . Results are presented as a function of T_e for $N_{\text{Cs}}^0 = 3 \times 10^{22} \text{ m}^{-3}$, (typical values for certain thermionic diodes). Both sets of results are compared with the corresponding Saha curves for plasmas in local thermodynamic equilibrium (LTE). A discussion of rules for choosing reasonably accurate but simple atomic models is included. Those electronic levels which serve as free electron source terms are identified as functions of N_{Cs}^0 for fixed T_e . Even a 5-level model gives N_e values which are only in fair agreement with optically thick results for a 26-level model under certain conditions. The lumped level degeneracy g_M can be made a function of N_{Cs}^0 and T_e to bring the $M = 5$ results into better agreement with the more reliable $M = 26$ results of Norcross and Stone. However, in the absence of such a comparison it does not appear that construction of accurate but simple atomic models is a straightforward process.

INTRODUCTION

There is much interest in the accurate calculation of free electron number density N_e in nonequilibrium plasmas of low electron temperature T_e that exist in Cs plasma diodes. The purpose of this study is to arrive at rules for constructing simple but satisfactory model atoms. Such models must ensure reasonably accurate values of N_e for the aforementioned plasmas as well as allow simplified calculations of the free electron distribution function. The accuracy of such models depends upon the number and arrangement of electronic energy levels as well as inelastic cross sections, radiative lifetimes and radiative capture coefficients. For the steady state plasmas of interest, the free electrons are assumed to have a Maxwellian distribution. Solutions of the Boltzmann equation (including all elastic and inelastic collision terms) along the lines described in Ref. 1 indicate that the distribution function $f_e(u)$ is Maxwellian down to T_e values of 2100°K (for Cs plasmas optically thick to resonance radiation). Although the numerical method described in Ref. 1 was convergent only for $T_e \geq 2300^\circ \text{K}$, subsequent calculations down to $T_e = 2100^\circ \text{K}$ have been found to yield essentially Maxwellian $f_e(u)$. The high energy tail of $f_e(u)$ may be non-Maxwellian for T_e values $< 2100^\circ \text{K}$ but the N_e values can be relatively insensitive to this tail ($u > 3.89 \text{ eV}$) for optically thick plasmas. This insensitivity is present where ground state ionization becomes a relatively unimportant rate process.

DETAILS OF THE CALCULATION

A model atom consisting of M levels is studied. The rate of change of the number of bound electrons in any level L is¹:

$$\dot{N}_L = -N_L(N_e \mathcal{K}_L + \mathcal{A}_L) + N_e \sum_{K \neq L} N_K K_{K \rightarrow L} + \sum_{K > L} N_K A_{K \rightarrow L} + N_e N_i (K_L^{\text{cap}} N_e + \beta_L) \quad (1)$$

In Eq. (1) $K_{L \rightarrow K}$ ($\text{m}^3 \text{sec}^{-1}$) is the excitation coefficient for the collision induced transition $L \rightarrow K$ and $K_{K \rightarrow L}$ ($\text{m}^3 \text{sec}^{-1}$) is the corresponding coefficient for de-excitation. The quantities K_L^{cap} ($\text{m}^6 \text{sec}^{-1}$) and β_L ($\text{m}^3 \text{sec}^{-1}$) are the three-body and radiative capture coefficients, respectively. \mathcal{K}_L ($\text{m}^3 \text{sec}^{-1}$) is the total collision coefficient for transitions out of state L ; i.e. $\equiv \sum_{K \neq L} K_{L \rightarrow K}$

+ K_L^{ion} where K_L^{ion} ($\text{m}^3 \text{sec}^{-1}$) is the ionization coefficient for level L . $A_{K \rightarrow L}$ (sec^{-1}) is any radiative transition probability for de-excitation into the L^{th} state and \mathcal{A}_L is the sum of the $A_{L \rightarrow K}$ (sec^{-1}) for radiative transition to all levels below L .

For the steady state treated here $\dot{N}_L = 0$ and the number densities must satisfy the plasma normalization condition:

$$N_i + \sum_{L=1}^M N_L = N_{\text{Cs}}^0 \quad (2)$$

Equations (1) and (2) together with the condition of charge neutrality ($N_e = N_i$), serve to determine N_e and the N_L values for a specified $f_e(u)$ and initial Cs number density N_{Cs}^0 . Equation (2) can be rewritten as:

$$f + \sum_{L=1}^M N_L' = 1 \quad (2a)$$

where f is the ionization fraction and N_L' is the normalized (to N_{Cs}^0) population of the L^{th} state. In the steady state the total ionization rate from bound levels must balance the total capture rate of free electrons into all such levels. This relation provides a useful check on the numerical calculations. Its application indicated that accurate numerical solutions for f were not possible at low values of N_e without tight convergence criteria.

The condition, $N_e = 0$ can be written as:

$$\sum_{L=1}^M [(K_L^{\text{cap}})^* f^2 + \beta_L f] = \sum_{L=1}^M K_L^{\text{ion}} N_L' \quad (3)$$

where $(K^{\text{cap}})^*$ ($\text{m}^3 \text{sec}^{-1}$) is an effective two-body capture coefficient $\equiv N_{\text{Cs}}^0 K_L^{\text{cap}}$. Thus the equation for the ionization fraction is essentially quadratic; however, the normalized populations N_L' are strong functions of N_{Cs}^0 and optical thickness for fixed T_e . The ionization coefficient $K_L^{\text{ion}} \equiv \langle Q^{\text{ion}}(v_e) v_e \rangle$ is of course a function of T_e alone.

MODEL ATOM PROPERTIES

The optimization of the model atom depends upon the choices of cross sections for electron impact and coefficients and frequencies for radiative processes involving the M^{th} level. This level must be adjusted so as to simulate the presence of many missing excited levels¹. The assignments of ionization potential E_M statistical degeneracy g_M and \mathcal{A}_M and β_M values thus become critical for accurate calculations of f values. The N_M' for this lumped level is a complicated function of the level structure for variable N_{Cs}^0 (see Eq. (1)). The schematic diagram for the three models is shown in Fig. 1 with values of E_L , $A_{L \rightarrow K}$ and g_L .

The early form of the Gryzinski cross sections has been used to compute ionization coefficients and excitation coefficients for optically allowed transitions only.² The Gryzinski exchange formulation was used to compute excitation coefficients for optically forbidden transitions.³ Although

the exchange cross sections have relatively small values at maximum, their slopes are steep just above threshold. Since the excitation and ionization thresholds are usually $\gg kT_e$, the behavior of the cross section in this region determines the value of the excitation coefficient. The lumped level was treated as an optically allowed state for all excitation (and de-excitation) collisions. Most allowed and exchange cross sections (monoenergetic) give reasonable agreement with experiment over a range of electron energies from threshold to maximum⁴. The experimental value of the most important excitation cross section $Q_{1 \rightarrow 2}^{\text{ex}}$ has a slope roughly 3 times the Gryzinski value at threshold⁴.

The radiative capture coefficients β_L were taken from Ref. 5 where they were calculated using an adjusted quantum defect method. These calculations agree well with known oscillator strengths and recombination cross sections.

The radiative transition probabilities were taken from Ref. 6; the agreement of those calculations with cited experimental values is within 50% for all important A_{LK} values (see Ref. 7). The most critical A_{LK} value is for the $2 \rightarrow 1$ resonance transition; this line is strongly absorbed for all experimental plasmas of interest⁸. For $M = 3$ the A_{32} value was set equal to the sum of A_{52} , A_{42} , and A_{32} . The value of A_{21} was used for all three optically thin cases.

RESULTS - OPTICALLY THIN PLASMAS

Since the only available $M = 26$ results (optically thin) are several points for $T_e = 3000^\circ \text{K}$, the $M = 2$ and 3 results will be compared with 5-level results for those plasmas. Since ground state ionization is a significant process at low N_{Cs}^0 values, however, a non-Maxwellian tail in $f_e(u)$ could cause large changes in calculated values of f in this region.

Low N_{Cs}^0 limit. - In the limit of low N_{Cs}^0 values three-body recombination is negligible and the free electron number density is determined by (from Eq. (3))

$$\sum_{L=1}^M \beta_L f \cong \sum_{L=1}^M K_L^{\text{ion}} N_L^1 \quad (4)$$

In the optically thin case, radiative capture into excited states is the important capture process which balances ground state ionization. Then f is given approximately by:

$$f \cong K_1^{\text{ion}} N_1^1 / \sum_{L=2}^M \beta_L \quad (4a)$$

which simplifies for $M = 2$. The product $K_1^{\text{ion}} N_1^1 \cong K_1^{\text{ion}}$ (for all M), however, since the ground state population is greater than $0.95 N_{Cs}^0$ for optically thin plasmas in the low N_{Cs}^0 regime. The value of f is then simply determined by the ratio of the ground state ionization coefficient to the sum of the largest radiative capture coefficients. For cesium the β_3 (5D level) value dominates

the $\sum_{L=2}^M \beta_L$ term; it is three times the maximum β value for other excited levels, $\beta(6P)$, and 200 times the β_1 (6S) value⁵. Thus, in choosing simple model atoms, it is important to assign a large value of β_M to simulate the missing excited levels at low N_{Cs}^0 . The β_5 assigned for $M = 5$ is the sum of β values for the six excited states: 7P, 6D, 8S, 4F, 8P, 7D⁵.

The results of f versus N_{Cs}^0 for 2-, 3-, and 5-level models are shown in Fig. 2 for an electron temperature of 3000°K (0.26 eV). The f values for $M = 2$ (6S and 6P levels) with $\beta_2 = \beta(6P) + \beta(5D)$ agree with the $M = 5$ results within 50% at low values of N_{Cs}^0 ($\leq 3 \times 10^{20} \text{ m}^{-3}$). This agreement can be improved by adding the sum $(\beta(7S) + \beta_5)$ to the β_2 value. For a given excitation cross section $Q_{L \rightarrow K}^{\text{ex}}$ the de-excitation coefficient $K_{K \rightarrow L}$ is inversely proportional to the upper state degeneracy g_K . The L^{th} level 3-body capture coefficient is directly proportional to the g_L value. The change of degeneracy for the M^{th} state ($M = 2, 3$) does not affect

the f value at low N_{CS}^0 because the value of N_1^0 is unchanged. However, the degeneracy does affect the maximum ionization fraction calculated for the $M = 2$ and 3 models. This latter behavior can be described in terms of the free electron source terms although there is no simple limiting expression. The degree of departure of f from LTE conditions is evident by direct comparison with the Saha curve in Fig. 2.

High N_{CS}^0 limit. - In the limit of high N_{CS}^0 values 3-body capture becomes the dominant loss mechanism for free electrons. Radiative capture is negligible and the value of f can be approximated by:

$$f \cong \left[\frac{\sum_{L=1}^M K_L^{\text{ion}} N_L^0}{\sum_{L=1}^M (K_L^{\text{cap}})^*} \right]^{1/2} \quad (3)$$

The excited levels have the largest ionization coefficients since the ionization cross section Q_L^{ion} is a strong inverse function of ionization potential E_L . These levels are most nearly in equilibrium with the free electrons and f approaches the Saha result. For the simplest model atom, $M = 2$, one must choose values of g_M and E_M so as to preserve the "correct" ratio of ionization rate to 3-body capture coefficient. In this study the value of E_2 was fixed at the 6P value, 2.43 eV, and the value of g_2 was varied for best agreement with $M = 5$ results. However, it will be shown that the M^{th} state for $M = 5$ is not so simply adjusted to match the $M = 26$ results.

For a given ratio of ionization and capture coefficients for the lumped level N_2^0 must vary with N_{CS}^0 so as to account for the populations of the missing excited levels near the Saha limit. For $M = 2$, N_2^0 increases so rapidly ($\propto N_{CS}^0 0.95$) that $K_2^{\text{ion}} N_2^0$ exceeds $(K_2^{\text{cap}})^* f^2$ at intermediate values of N_{CS}^0 . The f is relatively small and only a gradual function of N_{CS}^0 so the 3-body rate does not become important for the optically thin case until high N_{CS}^0 values. The maxima in the f curves of Fig. 2 occur approximately where the collisional lifetime $= (K_2 N_e)^{-1}$ equals the radiative lifetime $(A_{21})^{-1}$ for the first excited state (see Ref. 9).

The $M = 3$ and 5 cases are similar in that net ionization from excited states is a significant process for $N_{CS}^0 > 10^{22} \text{ m}^{-3}$. The $M = 2$ model is unique because the ground state always serves as the free electron source term (i.e. exhibits net ionization). Thus in this case the lumped level necessarily becomes a net capturer since $\dot{N}_e = 0$. The large β_2 value (60 times β_1) ensures that this level remain a net capturer at low values of N_{CS}^0 . The sources of ionization for the $M = 3$ and 5 models can be determined from the net ionization rates. The rate for the L^{th} state can be written from Eq. (3) as:

$$R_L^{\text{net}} = K_L^{\text{ion}} N_L^0 - [(K_L^{\text{cap}})^* f^2 + \beta_L f] \quad (6)$$

Only R_L^{net} values which are positive will be considered, i.e., states which are free electron source terms. The variation of such source terms (normalized to the sum of positive terms) R_L^0 is shown for $T_e = 3000^\circ \text{ K}$, optically thin, in Fig. 3 for $M = 5$ and 3. The $L = 4$ level (7S) makes the largest contribution to the free electron population at $T_e = 3000^\circ \text{ K}$ for $M = 5$, $N_{CS}^0 > 2 \times 10^{21} \text{ m}^{-3}$. This behavior differs slightly from the $T_e = 0.2 \text{ eV}$ results where the $L = 3$ state (5D) has the largest R_L^0 value at high N_{CS}^0 .

The results for $M = 5$, $g_5 = 50$, $E_5 = 0.6 \text{ eV}$ of Fig. 2 are in good agreement (within a factor of two) with $N_e/(N_e)_{\text{Saha}}$ results for the 26-level model at $T_e = 3000^\circ \text{ K}$ (see Table I). No optically thin results for $M = 26$ were available for comparison at lower T_e values. The simpler models should give good agreement with the $M = 26$ case at low N_{CS}^0 ($\leq 10^{19} \text{ m}^{-3}$) since the lumped β_M values have been chosen so as to produce that behavior.

RESULTS - OPTICALLY THICK PLASMAS

Low N_{CS}^0 limit. - At low values of N_{CS}^0 ($< 10^{19} \text{ m}^{-3}$ in Fig. 4(a)) f is determined by the balance between ionization from excited states and radiative capture. Just as in the optically thin

case for high N_{CS}^0 , the excited states are free electron source terms, but at low N_{CS}^0 . The dominant R_L^i is R_2^i for $N_{CS}^0 \leq 10^{19} \text{ m}^{-3}$ in the 3- and 5-level models. Only the ground state has a positive R_L^{net} value for $M = 2$ through the range of N_{CS}^0 values. The approximate relation for the ionization fraction becomes:

$$f \cong \frac{\sum_{L=2}^M (K_L^{\text{ion}} N_L^i)}{\sum_{L=2}^M \beta_L} \quad (6)$$

For $M = 5$ the above ratio should give reasonably accurate f values when just the $L = 2$ and 3 ionization terms and the β_3 value are included. However, at intermediate values of N_{CS}^0 3-body capture is comparable with the radiative process and the lumped level must represent the missing excited states. The excited states are free electron source terms at low N_{CS}^0 because their populations (especially N_2^i) are enhanced in the absence of radiative de-excitation. The ionization rates become large even when populations N_L^i , $L > 1$ remain $\approx 10^{-2} \rightarrow 10^{-3}$ of N_1^i because the ionization coefficients $K_L^{\text{ion}} \gg K_1^{\text{ion}}$. The monoenergetic cross sections Q_L^{ion} , $L > 1$, have lower energy thresholds and much higher slopes than Q_1^{ion} in the threshold region. The excitation coefficients for collision-induced transitions between excited levels are correspondingly larger than the ionization coefficients and have low thresholds (some $< 1 \text{ eV}$).

The f values for the optically thick case are plotted versus N_{CS}^0 for $T_e = 3000^\circ \text{ K}$ in Fig. 4(a). The lumped β_M value should guarantee accuracy of the simpler models at low N_{CS}^0 . This is true if the populations N_2^i have nearly the same values as for $M = 26$. Decreasing β_M for $M = 2$ gives better agreement with $M = 26$ at $N_{CS}^0 = 10^{20} \text{ m}^{-3}$ but the asymptote is incorrect at 10^{18} m^{-3} . For $M = 3$, the f value is increased at all N_{CS}^0 by making the plasma thick to all line radiation and lowering g_M from 50 to 10. There is no peak in f versus N_{CS}^0 for $M = 2$, $g_2 = 50$ since the $L = 2$ state has a large 3-body capture rate at intermediate N_{CS}^0 due to its large g_M value and relatively high N_e . Lowering the degeneracy to $g_2 = 6$ (not shown) does not change the curve shape since 3-body capture continues to dominate excited state ionization. Decreasing the g_M from 50 (curve 4) to 10 (not shown) for $M = 3$ does produce a maximum in f because $3 \rightarrow 2$ de-excitation is increased enough to raise the N_2^i value significantly. This $L = 2$ state is the main free electron source term for $M = 3$.

The f values for the optically thick case, $T_e = 2321^\circ \text{ K}$, are plotted versus N_{CS}^0 in Fig. 4(b). Just as for $T_e = 3000^\circ \text{ K}$ the f is determined by ionization from the ground level for $M = 2$ and from excited states for $M = 3$ and 5. These ionization processes balance radiative capture into all levels and the curves remain flat where the populations of the ionization sources remain constant, just as for 3000° K . The features of the curves are identical to the $T_e = 3000^\circ \text{ K}$ results throughout the range of N_{CS}^0 values. The peak for $M = 5$ is again caused by ionization from the higher excited states $L = 3$ and 4. Four points of $M = 26$ results are also shown in Fig. 4(b). Comparison with the latter results indicate that the simpler atomic models overestimate departures from LTE at values of N_{CS}^0 below $3 \times 10^{22} \text{ m}^{-3}$. It should be noted, however, that the N_e values from $M = 26$ are considerably lower than Saha values for $N_{CS}^0 \leq 10^{21} \text{ m}^{-3}$. The ionization fractions are approximately 10^{-2} of the 3000° K values because excited state populations and ionization coefficients are considerably lower. The limiting (low N_{CS}^0) value of f for $M = 3$ is only 10^{-2} of the f computed for a 3-level model in Ref. 9. For $M = 2$ the value of f is about $1/15$ of that for a 2-level model in Ref. 9. These large differences are due to the present use of more reliable excitation cross sections and \mathcal{A}_L and β_L values.

The variation of the free electron source terms for $M = 5$ at $T_e = 2321^\circ \text{ K}$ (0.2 eV) and 3000° K are shown in Figs. 5(a) and 5(b), respectively. Qualitatively, the dependences of R_L^i are similar but the region of rapidly changing contribution is shifted by a factor of 10^2 to higher N_{CS}^0 values at the lower value of T_e . The $M = 26$ results of Ref. 7 indicate that the maximum f is nearly 5 times the $M = 5$ value for $N_{CS}^0 \approx 10^{21} \text{ m}^{-3}$, $T_e = 2321^\circ \text{ K}$. Figure 5(b) shows that the main contributor of free electrons is the $L = 3$ state for $N_{CS}^0 > 3 \times 10^{20} \text{ m}^{-3}$ and the ground and first excited states for $N_{CS}^0 \leq 3 \times 10^{20} \text{ m}^{-3}$. If additional excited states are to become

new ionization source terms, their relative populations must fall off slowly enough with binding energy that their higher ionization coefficients can raise f . The sudden drop in the $L = 3$ contribution in the region $N_{CS}^0 \leq 10^{21} \text{ m}^{-3}$ is a result of the large β_3 value since radiative capture becomes the dominant electron loss process in that region ($N_e \leq 5 \times 10^{18} \text{ m}^{-3}$).

The agreement between the $M = 5$ and 26 results improve at high $N_{CS}^0 (\approx 10^{22} \text{ m}^{-3})$ for $T_e = 3000^\circ \text{ K}$ as can be seen in Fig. 4(a). The f for $M = 5$ is about 90% of the f value for $M = 26$. Also, the disparity near peak ($N_{CS}^0 \approx 10^{20} \text{ m}^{-3}$) is explainable in terms of the R_L' value just as the $T_e = 2321^\circ \text{ K}$ results. Figure 5(b) shows that the $L = 4$ level is the chief contributor of free electrons for $N_{CS}^0 > 3 \times 10^{19} \text{ m}^{-3}$. This occurs because the "ladder" ionization mechanism operates if the excited state populations are large enough. For optically thick plasmas this is the case. It is plausible, therefore, that including additional excited states would raise the value of f in this T_e region if these states serve as large source terms with R_L' values $\approx R_4'$.

High N_{CS}^0 limit. - At high values of N_{CS}^0 the excited levels are still the free electron source terms. Again the balance which determines f is between excited state ionization and three-body capture into these same levels. The f values, optically thick, are uniformly larger at lower N_{CS}^0 values than in the optically thin case. This is because the important N_L' approach their Saha values at lower $N_{CS}^0 (\approx 3 \times 10^{21} \text{ m}^{-3})$. The value of f can be satisfactorily calculated from Eq. (5) including terms, $L = 2$ to M .

Results for $T_e = 1740^\circ \text{ K}$ (0.15 eV). - Curves of f versus N_{CS}^0 were also calculated at $T_e = 1740^\circ \text{ K}$. There is no structure in these curves for $N_{CS}^0 < 10^{25} \text{ m}^{-3}$, optically thin and $N_{CS}^0 < 10^{26} \text{ m}^{-3}$, optically thick. One point was compared with the $M = 26$ results at $N_{CS}^0 = 10^{24} \text{ m}^{-3}$, optically thick. The $M = 26$ model gives an f which is 3 times the value for $M = 5$, $g_5 = 50$. Since the chief free electron source term is $L = 5$ for $M = 5$ the difference between the $M = 5$ and 26 results becomes very sensitive to the lumped state. This level must now simulate the source terms for N_e .

The simpler models do not agree well with each other at $T_e = 0.15 \text{ eV}$, optically thick. This behavior is shown in Fig. 6 where f is plotted as a function of T_e at $N_{CS}^0 = 3 \times 10^{22} \text{ m}^{-3}$ for $M = 2, 3$, and 5. The largest disparity occurs in the optically thick case where the f for $M = 2$ is only 2% of the value calculated for $M = 5$. This result is consistent with the fact that the former model does not allow for excited state source terms. The $M = 3$ model gives an f value midway between the $M = 2$ and 5 results. The values of f , optically thin, are only $\approx 10^{-3}$ of the optically thick values; this is because the ground state is the sole ionization term in the former case. Thus at this low T_e value the ionization mechanism changes from simple ground state ionization to a "stepwise" ladder process (for $N_{CS}^0 = 10^{18} \rightarrow 10^{25} \text{ m}^{-3}$) as the optical thickness changes. As the electron temperature is increased this disparity between optically thin and thick results decreases markedly. For fixed values of β_M and E_M , agreement with $M = 26$ results for simpler models can be improved by using variable $g_M = F(N_{CS}^0, T_e)$ or making the plasma thick to all radiation. However, any scaling laws for construction of atomic models remain to be investigated in the absence of more detailed results for comparison.

CONCLUDING REMARKS

For certain values of electron temperature and neutral density 2- and 3-level models can be adjusted to give N_e values which agree with results for 5-levels. However, for the experimentally interesting case of optically thick Cs plasmas, results for a 26 level model⁷ indicate the "best" $M = 5$ model can seriously underestimate the ionization fraction f . This is especially true for values of electron temperature $< 3000^\circ \text{ K}$. This disparity occurs because the sources of free electrons are highly excited levels as a result of a "ladder" ionization mechanism. It does not appear likely that convenient adjustment of simpler models will improve agreement with the $M = 26$ results. For proper choices of E_M , β_M , \mathcal{A}_L for the lumped level a variable $g_M(N_{CS}^0)$ will provide satisfactory agreement for fixed T_e . However, it appears that increasing the number of

levels is a more desirable approach than this to ensure calculation of accurate N_e values. Further results for $M = 26$ are needed to evaluate the simpler model atoms for the low temperature ($1740^\circ - 2321^\circ$ K) optically thin plasmas.

TABLE I. - ($T_e = 3000^\circ$ K; OPTICALLY THIN)

M	$N_e/(N_e)_{\text{Saha}}$			
5	2.4×10^{-2}	6.8×10^{-2}	0.35	0.75
26* (Ref. 7)	$\cong 1.3 \times 10^{-2}$	$\cong 6.5 \times 10^{-2}$	$\cong .32$	$\cong .75$

*Points estimated from plots without grid.

REFERENCES

1. J. V. Dugan Jr., F. A. Lyman and L. U. Albers, Preprint No. SM-74/191, International Atomic Energy Agency, Vienna, (1966). (USAEC CONF-660704-60).
2. M. Gryzinski, Phys. Rev., 115, no. 2, 374 (1959).
3. M. Gryzinski, Phys. Rev., 138, no. 2A, A336 (1965).
4. J. V. Dugan Jr. and R. J. Sovie, Proposed NASA Technical Note.
5. D. W. Norcross and P. M. Stone, J. Quant., Spectros. Radiat. Transfer 6, 277 (1966).
6. T. G. Roberts and W. L. Hales, Army Missile Command, Redstone Arsenal AMC-RR-TR-62-8, (1962).
7. D. W. Norcross and P. M. Stone, Reports of 27th Annual Physical Electronics Conf., M.I.T. (1967).
8. M. A. Lutz, AIAA Jour., 3, no. 4, 772 (1965).
9. D. J. Ben Daniel and S. Tamor, GE Research Lab, Report No. 62-RL-2922E, (1962).

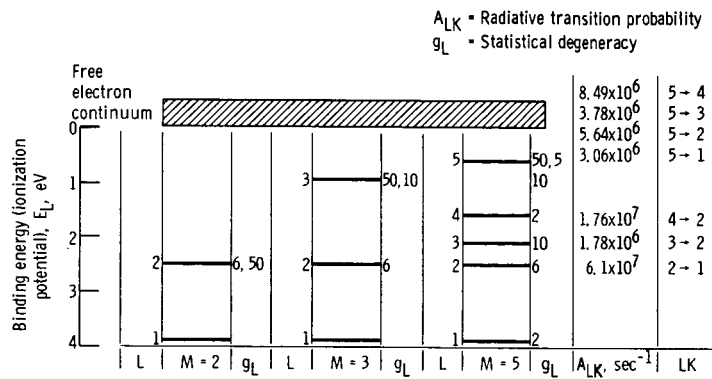


Figure 1. - Schematic diagram of cesium model atoms.

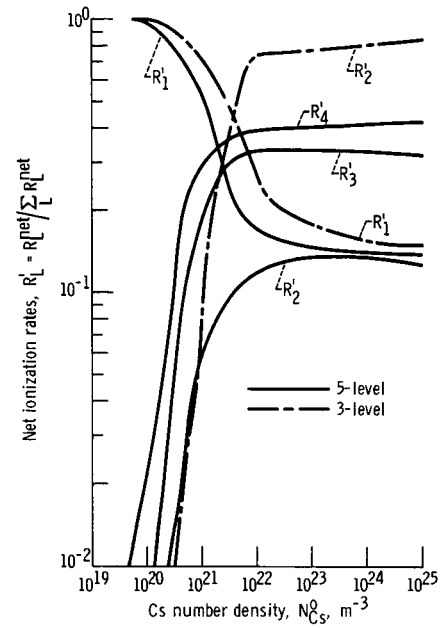
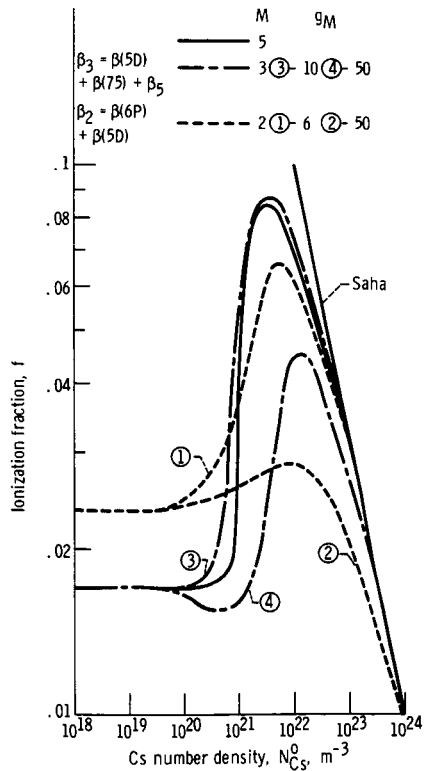
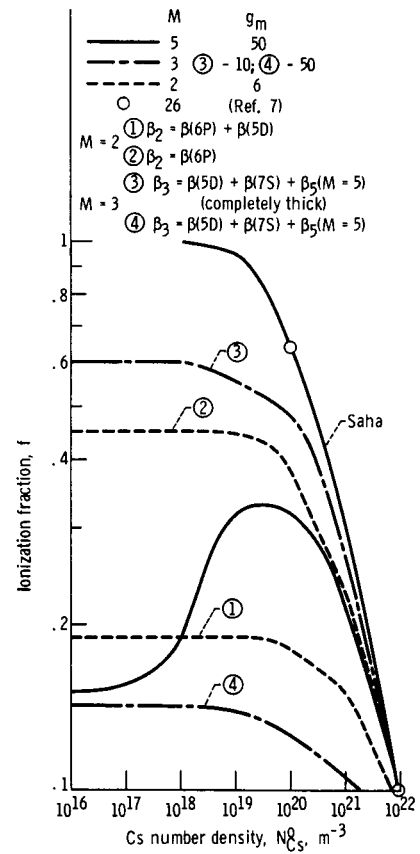
Figure 3. - Plots of normalized net ionization rates as a function of Cs neutral density in plasmas optically thin to resonance radiation for 3- and 5-level atomic models. $T_e = 3000^\circ \text{K}$.Figure 2. - Plots of ionization fraction as a function of Cs neutral density in plasmas optically thin to resonance radiation for 2-, 3-, and 5-level atomic models. $T_e = 3000^\circ \text{K}$.(a) $T_e = 3000^\circ \text{K}$. Two points for a 26-level model are shown for comparison.

Figure 4. - Plots of ionization fraction as a function of Cs neutral density in plasmas optically thick to resonance radiation for 2-, 3-, and 5-level atomic models.

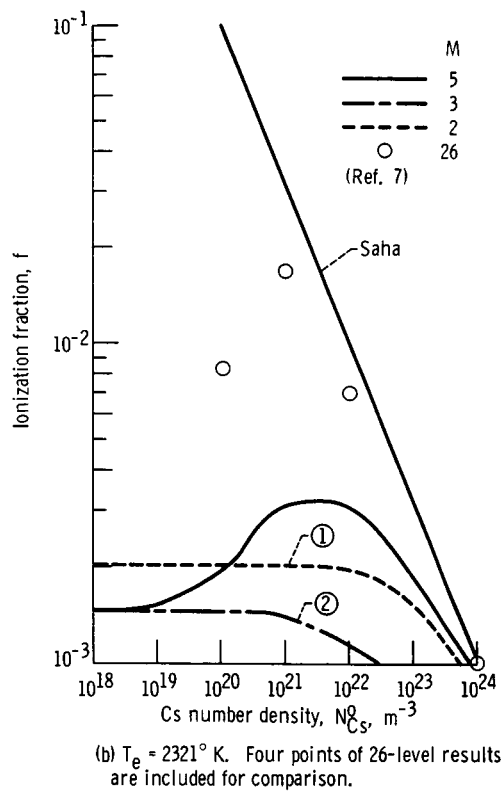


Figure 4. - Concluded.

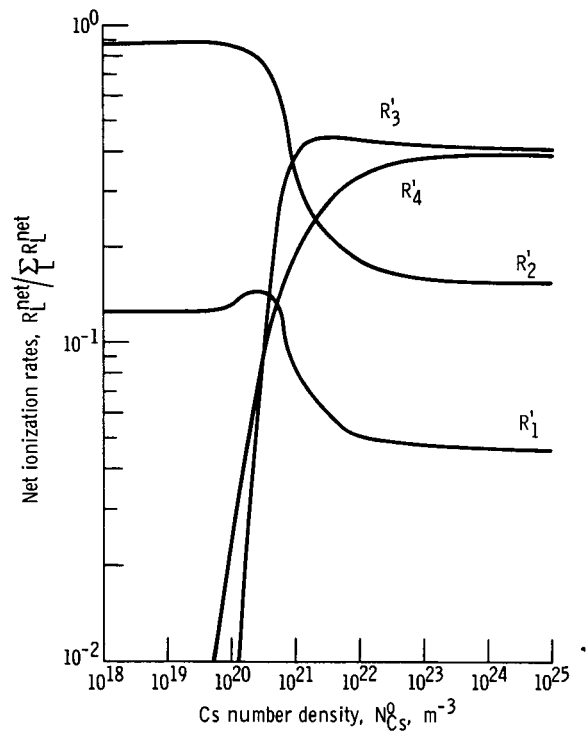


Figure 5. - Concluded.

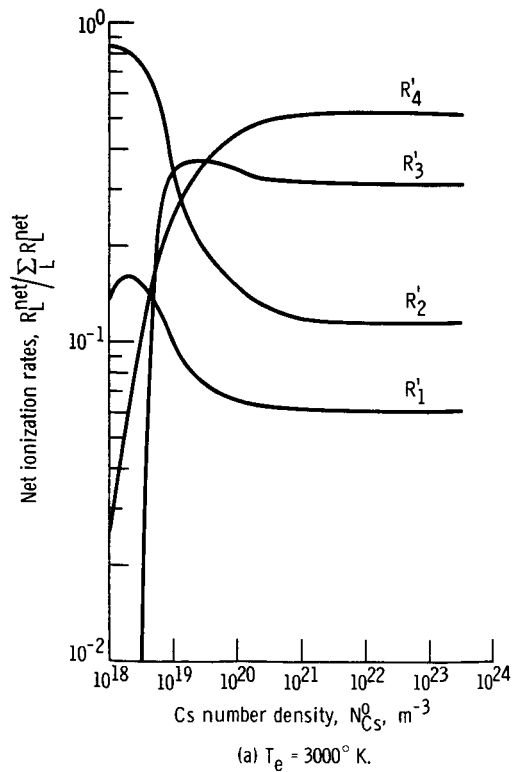
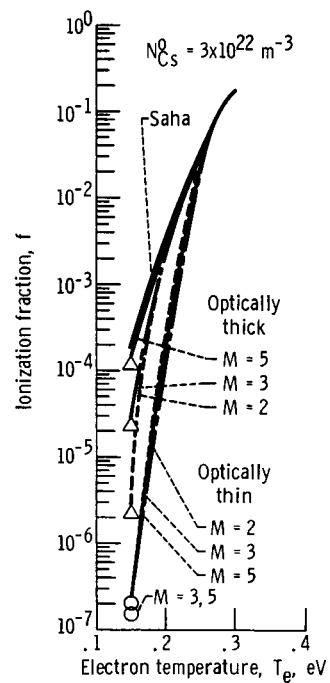


Figure 5. - Plots of normalized net ionization rates as a function of Cs neutral density in plasmas optically thick to resonance radiation for the 5-level atomic model.

Figure 6. - Plots of ionization fraction as a function of electron temperature in plasmas optically thin and thick to resonance radiation for $M = 2$ -, 3 -, and 5 -level atomic models.